CONVERSION OF ELECTROMAGNETIC TO ACOUSTIC ENERGY

Antonio Manuel Machado Dos Santos Reto

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THESIS

CONVERSION OF ELECTROMAGNETIC
TO ACOUSTIC ENERGY

by

Antonio Manuel Machado Dos Santos Reto

December 1976

Thesis Advisor: O.M. Baycura

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A discussion of the experimental apparatus and procedures used in an attempt to experimentally verify some of the results given in the theoretical formulation, is also made.

Conversion of Electromagnetic
to Acoustic Energy

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ABSTRACT

The conversion of electromagnetic to acoustic energy is a broad and complicated subject. Some topics can be found in the literature but its thorough treatment is rather complex.

Engineering approaches to the problem are however possible through useful simplified models. Simplified treatments of the three major mechanisms leading to the production of mechanical forces on materials supporting an electromagnetic field are to be presented. These mechanisms are referred to as radiation pressure, transient surface heating and stimulated Brillouin scattering.

A discussion of the experimental apparatus and procedures used in an attempt to experimentally verify some of the results given in the theoretical formulation, is also made.

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I. INTRODUCTION

A. NATURE OF THE PROBLEM

The generation of mechanical forces on materials supporting an electromagnetic field was first predicted several decades ago although its detailed theoretical treatment had been done much more recently. The literature concerning this phenomenon is not plentiful and is rather complex. Since the efficiency of the energy conversion is extremely low, the experimental verification of the theoretical predictions was only possible after high power electromagnetic sources became available. Radiation from powerful radars as well as from high power pulsed lasers are the most suitable sources of electromagnetic energy for the study of the several mechanisms involved in this conversion process.

The possible interpretations of the mechanical forces produced in a material medium subjected to interaction with an electromagnetic field, may be divided into three major types. That is, the forces may appear due to one or more of the following mechanisms:

(i) radiation pressure,

(ii) transient surface heating,

(iii) stimulated Brillouin scattering.

It is a familiar fact that electromagnetic waves carry energy through the medium where they propagate in accordance

with Poynting's vector. Less familiar, however, is the fact that electromagnetic waves may also transport linear momentum.

Originally, the term radiation pressure was used to designate the pressure exerted on an object by shining light on it. This was predicted theoretically by Maxwell about 1870 but the first measurement of radiation pressure was made latter in 1901-1903. According to Maxwell's predictions, if a parallel beam of light carrying the energy U falls on an object for a time t and is entirely absorbed, the magnitude of the momentum $\vec{p}$ delivered to it is given by $p = U/c$, where c is the speed of light. The direction of $\vec{p}$ is the direction of the incident beam. When the energy U is entirely reflected, the magnitude of the momentum delivered is twice that given above, or $p = 2U/c$. On the other hand, if the energy U is partly reflected and partly absorbed, the delivered momentum will lie between U/c and $2U/c$.

The resultant forces are so small in relation to forces of the daily experience that they cannot be ordinarily noticed.

Nowadays radiation pressure has a much broader significance. It is interpreted as the mechanism responsible for the production of forces in a medium due to a rate of change of electromagnetic momentum associated with moving energy as its equivalent mass. This rate of change of electromagnetic momentum is the consequence of energy gradients

established in the medium due to conversion of energy to heat or to progressive changes in the storage energy; it can also be the consequence of the existance of velocity gradients as it is the case when part of the incident energy is reflected back at the interface between two different media.

The radiation pressure forces are in general much smaller than the forces produced through the transient heating of the surface of a medium.

The generation of acoustic waves utilizing the thermal effects accompanying the absorption of electromagnetic radiation by an elastic medium was first experimentally verified in 1881 and since then just a few theoretical and experimental works have been developed. The subject seems to be receiving now more attention probably due to the fact that the recent advances in high power laser technology gives to this process the possibility of concrete and useful practical applications.

When a pulse of electromagnetic radiation passes through an absorbing medium, its intensity is exponentially reduced producing a rise in the temperature of the medium. The thermal shock occuring as a consequence of the intense surface heating produces a thermal expansion originating an acoustic wave which propagates away from the heated surface. That is, a simple thermoacoustic source is established in the medium.

It is possible to extend this concept of a simple thermoacoustic source to the one of a thermoacoustic array which may be produced in the medium when it is subjected to a train of periodic pulses of high energy. In this case, a highly directional acoustic wave is launched from the heated region and propagates in a direction normal to the electromagnetic beam.

As suspected, the analysis of this thermodynamic conversion process involves the knowledge of theoretical principles related to fields like solid and fluid mechanics and thermodynamics.

So far, the mechanisms leading to the conversion of electromagnetic to acoustic energy were described by an interaction process between an electromagnetic wave and the matter supporting it. Under certain conditions, sound may also result when two intersecting laser beams of very high energy interact with a medium. If the angle of interaction of the two beams is the appropriate, optically induced generation and amplification of sound will be verified. This is a particular case of the scattering phenomenon and is called stimulated Brillouin scattering. Note that this is not the same mechanism as the transient surface heating. While the latter utilizes the thermalization of modulated electromagnetic energy, the former is based on the mechanics of travelling wave parametric interactions where the resultant acoustic wave has a frequency equal to the difference in the frequencies of the two interacting light beams.

B. BACKGROUND CONCEPTS

A more detailed theoretical analysis of the major mechanisms responsible for the conversion process will be presented next. But before that, it seems to be worthwhile to make a brief but hopefully clarifying comment about some unfamiliar terms belonging to fields like tensor calculus, mechanics and thermodynamics. The terms to be introduced here are mentioned in the theoretical formulation to be presented.

Tensor analysis — this is a rather advanced subject in mathematics and so just a few basic concepts will be mentioned.

In a three dimensional system consider two vectors $\vec{A}$ and $\vec{B}$ such that each rectangular component of vector $\vec{B}$ is a linear function of the components of vector $\vec{A}$, that is

$$B_1 = T_{11}A_1 + T_{12}A_2 + A_{13}A_3 ,$$
$$B_2 = T_{21}A_1 + T_{22}A_2 + T_{23}A_3 ,$$
$$B_3 = T_{31}A_1 + T_{32}A_2 + T_{33}A_3 .$$

If the association of the components of vector $\vec{B}$ with those of $\vec{A}$ is to be maintained for any rotation of the system coordinates, then the coefficients T_{ij} have to transform in a specific manner. By definition, a tensor of rank two is a linear transformation of the components of a vector $\vec{A}$ into the components of a vector $\vec{B}$ which is invariant to

rotations of the coordinate system. The nine components T_{ij} of the linear transformation are invariant scalars and are called the tensor components.

The divergence of a tensor of second rank is a vector, or tensor of first rank, and the divergence of a vector is an invariant scalar, or tensor of zero rank. The concept of a tensor can, of course, be generalized to include tensors of arbitrary rank.

Stress - suppose that a cut perpendicular to the x axis is made in a body in equilibrium. In order to maintain the equilibrium, an internal force would have to act in the exposed face. By definition, the stress at that point is the intensity of such a force, that is, the force per unit area. If $\Delta\vec{F}$ is the resultant contribution of these internal forces on an element of area ΔA, then the components of $\Delta\vec{F}$ along the three coordinate axis, ΔF_x, ΔF_y and ΔF_z, are called stress components and are defined by

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}, \quad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

σ_x is the normal stress and τ_{xy}, τ_{xz} are the shear stresses and from its definitions it is clear that normal and shear stresses are, respectively, the intensity of force perpendicular and parallel to the plane of the area element. Normal

stresses are termed tensile if they pull away from the cut; those pushing against the face are called compressive stresses.

Obviously, any deformation of a body due to external forces will give rise to internal forces, or stresses. The stresses may then be thought of as entities describing the state of the internal action of a body.

Imagine now, as an extension of this concept, that a cube of infinitesimal dimensions is isolated from a body and has its faces perpendicular to the coordinate axis. The stesses exposed by the cube are then the three normal stresses σ_x, σ_y and σ_z and the six shear stresses τ_{xy}, τ_{yx}, τ_{yz}, τ_{zy}, τ_{zx} and τ_{xz}. It can be shown [Ref. 1] that $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$ and $\tau_{yz} = \tau_{zy}$.

The stress components may be arranged into a matrix format representing the state of stress at a point:

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

This is called a second rank stress tensor.

Strain — the concept of strain is related to the deformations of a body. The normal strains in the axis directions, ε_x, ε_y and ε_z, are defined as the change in length per unit length of a line segment in the direction

under consideration. It follows from this definition that strain is a dimensionless quantity. Shear strains γ_{ij} may be also defined; they represent changes in angles between line segments. The subscripts in γ_{ij} have similar meanings to the subscripts for shear stresses.

The strain components can also be assembled into a second rank strain tensor.

It should be noted that stress and strain are closely related quantities.

Elasticity — is the property of a material which enables it to return to its original size and shape after the cause of a deformation has been removed.

Bulk modulus of elasticity 'B' — also called modulus of dilatation, is defined as the ratio of the change in pressure on a body Δp, to the resulting fractional change in volume $-\Delta V/V$, or $B = -V\Delta p/\Delta V$.

Specific heat 'C' — this is the heat capacity per unit mass of a body; heat capacity is the ratio of the heat ΔQ supplied to a body to its corresponding temperature rise ΔT. So, $C = \Delta Q/m\Delta T$.

Thermal conductivity 'K' — is a constant of proportionality relating the time rate of heat transfer dQ/dt across a certain area A and the corresponding temperature gradient dT/dx, or $dQ/dt = -KA\ dT/dx$.

Mechanical equivalent of heat 'J' — since heat is just another form of energy, any energy unit could be a heat unit. Hence, the mechanical energy equivalent of heat

energy, that is, the number of joules equivalent to one calorie is called mechanical equivalent of heat, $J = 4.185$ J/cal.

Coefficient of linear thermal expansion 'a' — the change in any linear dimension of a body is called a linear expansion. This coefficient is then a constant of proportionality relating the temperature change ΔT of a body, its original linear dimension ℓ and the corresponding change in this dimension $\Delta\ell$, or $\Delta\ell = a\ell\Delta T$.

Adiabatic process — is a process taking place in such a way that no heat flows into or out of the system. In practice, since the flow of heat is somewhat slow, any process can be made approximately adiabatic if it is performed quickly enough.

II. THEORY

A. RADIATION PRESSURE

1. Elastic Stress Tensor

The study and description of a certain category of physical phenomena like for example, volume deformations, cannot be done correctly by the use of the scalar or vector concepts. The adequate treatment of this type of problems requires the use of the tensor concept; tensor calculus, a somewhat unfamiliar tool, plays then an important role in the analysis of such problems.

When dealing with mechanical forces on a material medium, it is necessary to introduce an entity called elastic stress tensor. This is a tensor of second rank and so it has nine components, the stresses, representing the forces exerted on unit elements of area. It turns out that the divergence of this tensor is a vector, or tensor of first rank, giving the force per unit volume caused by the elastic stresses.

2. Electromagnetic Stress Tensor

An electromagnetic field is now considered; as usual in electromagnetic theory, its definition is based upon the four vectors $\vec{E}$, $\vec{B}$, $\vec{D}$ and $\vec{H}$ satisfying Maxwell's equations. $\vec{E}$ and $\vec{H}$ are, respectively, the electric and magnetic field vectors. The electric and magnetic flux density vectors, $\vec{D}$ and $\vec{B}$, are related to $\vec{E}$ and $\vec{H}$ by the

constitutive equations $\vec{D} = \varepsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$ where ε is the permittivity and μ is the permeability of the medium under consideration.

The existence of an electromagnetic field in a material medium leads to the definition of the so called electromagnetic stress tensor; its nine components T_{xx}, T_{xy}, T_{zz}, T_{yx}, etc. The electromagnetic stresses are defined [Ref. 2] by the following relations

$$T_{xx} = \frac{\varepsilon}{2}(E_x^2 - E_y^2 - E_z^2) + \frac{\mu}{2}(H_x^2 - H_y^2 - H_z^2)$$

similarly for T_{yy}, T_{zz}

$$T_{xy} = T_{yx} = \varepsilon E_x E_y + \mu H_x H_y$$

similarly for T_{yz}, T_{zx} (1)

The divergence of this tensor, div T, is a vector quantity whose components in cartesian coordinates are obtained according to the rule

$$(\mathrm{div}\ T)_x = \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z}$$

similarly for $(\mathrm{div}\ T)_y$ and $(\mathrm{div}\ T)_z$ (2)

Using Maxwell's equations it can be shown [Ref. 3] that

$$\text{div } T = \rho\vec{E} + \vec{J}\times\vec{B} + \varepsilon\frac{\partial}{\partial t}(\vec{E}\times\vec{B}) \tag{3}$$

where ρ and $\vec{J}$ are, respectively, volume charge density and current density.

As already mentioned, the divergence of the elastic stress tensor is interpreted as a force per unit volume; this important result can be extended to the case of the electromagnetic stress tensor so that its divergence may be interpreted as an alectromagnetic force per unit volume.

3. Electromagnetic Momentum

It is apparent that the terms $\rho\vec{E}$ and $\vec{J}\times\vec{B}$ in the right-hand side of eq. (3) have dimensions of a force per unit volume. For a medium with ρ and $\vec{J}$ throughout a volume V, these quantities represent then the forces exerted on unit volume elements of charge and current, that is, they are the well known electric and magnetic forces exerted on charges and currents. This means that a portion of the total force acting on the unit volume appears as a force on the matter within that volume. Therefore, the remaining force exerted on the unit volume should be associated with the term $\varepsilon\ \partial/\partial t\ (\vec{E}\times\vec{B})$.

As the dimensions of $\vec{B} = \mu\vec{H}$ are $MQ^{-1}T^{-1}$ and those of $\vec{D} = \varepsilon\vec{E}$ are QL^{-2}, the quantity $\varepsilon(\vec{E}\times\vec{B})$ is dimensionally

$QL^{-2}MQ^{-1}T^{-1} = ML^{-2}T^{-1} = MLT^{-1}L^{-3}$ or, a momentum per unit volume since the dimensions of momentum, $\vec{p} = m\vec{v}$, are MLT^{-1}. As a consequence, the term $\varepsilon(\vec{E}\times\vec{B})$ is interpreted as the electromagnetic momentum density. Now it is clear that the remaining force exerted on the unit volume by the external field acts by increasing the electromagnetic momentum within that volume.

The precise manner in which electromagnetic momentum density should be defined has found some differences of opinion. The existence of a duality of treatment of such concept results mainly from the two different ways in which matter can be visualized. On the one hand, a macroscopic presentation of a medium may be adopted when dealing with electromagnetic propagation in that medium; on the other, a microscopic picture describing matter as an arrangement of dipoles, doublets and charges surrounded by vacuum, exists associated with the concept of energy whose value depends on the relative velocity, but which always moves, regardless of the medium, with the velocity of light in vacuum.

According to the latter interpretation, the electromagnetic momentum density is defined [Ref. 4] by $\mu_o\varepsilon_o(\vec{E}\times\vec{H})$, where $c = 1/\sqrt{\mu_o\varepsilon_o}$ is the velocity of light in free space.

To summarize, it can be stated that associated with an electromagnetic field, there is a momentum distributed with a density $\vec{p}$ which is given, according to the specific interpretation, by one of the two possible expressions

$$\vec{p} = \mu_o \varepsilon_o (\vec{E} \times \vec{H}) \tag{4}$$

$$\vec{p} = \mu \varepsilon (\vec{E} \times \vec{H}) \tag{5}$$

Throughout this work the latter interpretation will be mostly used.

4. Forces in the Electromagnetic Field

The divergence theorem, a useful mathematical relation in electromagnetic theory,

$$\oint_S \vec{D} \cdot d\vec{S} = \int_V \nabla \cdot \vec{D} \, dv \tag{6}$$

states that the integral of the normal component of any vector field over a closed surface S, is equal to the integral of the divergence of this vector field throughout the volume enclosed by the closed surface.

This vector divergence theorem has a tensor analogue stating that the integral of the divergence of a tensor throughout a volume V is equal to the integral of its normal component over the surface S enclosed by V.

Integrating eq. (3) over the volume V and applying this theorem, the result would be

$$\oint_S T \cdot d\vec{S} = \int_V \rho \vec{E} \, dV + \int_V (\vec{J} \times \vec{B}) dV + \varepsilon \frac{\partial}{\partial t} \int_V (\vec{E} \times \vec{B}) \, dV \tag{7}$$

Consider for the moment a stationary distribution of charge and current; the fields are independent of time and so the last term on the right of eq. (7) is zero. The other two terms

$$\int_V \rho\vec{E}\, dV = \vec{F}_e \quad , \qquad \int_V (\vec{J}\times\vec{B})\, dV = \vec{F}_m \tag{8}$$

represent the net mechanical forces acting on the volume distributions of charge and current, where ρ and $\vec{J}$ are the charge and current densities within the closed surface S.

The total force transmitted by the electromagnetic field across the surface S or, the resultant force exerted on the charged matter within S is

$$\vec{F}_e + \vec{F}_m = \int_V (\rho\vec{E} + \vec{J}\times\vec{B})\, dV \tag{9}$$

It can be shown [Ref. 3] that

$$\vec{F}_e = \int_V \rho\vec{E}\, dV = \int_S [\varepsilon(\vec{E}\cdot\vec{n})\vec{E} - \frac{\varepsilon}{2} E^2\vec{n}]\, dS \tag{10}$$

$$\vec{F}_m = \int_V (\vec{J}\times\vec{B})\, dV = \int_S [\frac{1}{\mu}(\vec{B}\cdot\vec{n})\vec{B} - \frac{1}{2\mu} B^2\vec{n}]\, dS \tag{11}$$

where $\vec{n}$ is the outward unit normal at a point in the surface enclosing V, $E^2 = E_x^2 + E_y^2 + E_x^2 = |\vec{E}|^2$ and $B^2 = B_x^2 + B_y^2 + B_z^2 = |\vec{B}|^2$.

By Newton's second law

$$\vec{F}_e + \vec{F}_m = \frac{d}{dt}\vec{P}_m \tag{12}$$

$\vec{P}_m$ is the total linear momentum of the charged matter within S.

Consider now that the volume V contains no charges or currents. If the field is stationary, the net force across S is zero. But if the field is variable, the third term in the right-hand side of eq. (7) is not zero and so this equation becomes

$$\oint_S T \cdot d\vec{S} = \frac{\partial}{\partial t} \int_V \varepsilon(\vec{E} \times \vec{B})\, dV \tag{13}$$

The electromagnetic momentum concept shows up now to help in understanding this apparent action of a force arising in neutral matter immersed in an electromagnetic field. As stated before, $\varepsilon(\vec{E}\times\vec{B}) = \vec{p}$ is the density of the electromagnetic momentum associated with the electromagnetic field. The integral

$$\int_V \vec{p}\, dV = \vec{P}_e \tag{14}$$

is the total momentum of the field contained within V and then, as eq. (13) says, the force transmitted across S goes to increase the electromagnetic momentum of the field within V.

Consider at last a system composed of charges and variable fields within a bounded region. The conservation of momentum theorem is, therefore, according to eq. (7) expressed by

$$\frac{d}{dt}(\vec{P}_m + \vec{P}_e) = \oint_S T \cdot d\vec{S} \tag{15}$$

It follows from eqs. (9), (10) and (11) that

$$\vec{F} = \int_S [\varepsilon(\vec{E}\cdot\vec{n})\vec{E} + \mu(\vec{H}\cdot\vec{n})\vec{H} - \frac{1}{2}(\varepsilon E^2 + \mu H^2)\vec{n}]\, dS \tag{16}$$

This is the resultant force on the charge, current and matter within the surface S.

Equation (16) is a valid relation for both stationary and dynamic situations and the validity of its application to the case of variable fields can be justified if basic principles of the theory of relativity are invoked. Then, the rate of change of both mechanical momentum of the matter within S and electromagnetic momentum of the field within V, may be equated to the right-hand side of eq. (16) which must now be interpreted as the inward flow of momentum through the surface S per unit of time [Ref. 3]

$$\frac{d}{dt}(\vec{P}_m + \vec{P}_e) = \int_S [\varepsilon(\vec{E}\cdot\vec{n})\vec{E} + \mu(\vec{H}\cdot\vec{n})\vec{H} - \frac{1}{2}(\varepsilon E^2 + \mu H^2)\vec{n}]\, dS \tag{17}$$

Transforming this surface integral into a volume integral and using Maxwell's equations, the resultant expression would have the form

$$\frac{d}{dt}(\vec{P}_m + \vec{P}_e) = \int_V [\rho\vec{E} + \vec{J} \times \vec{B} - \frac{1}{2}(E^2 \nabla\varepsilon + H^2 \nabla\mu) + \frac{\partial}{\partial t}(\vec{D} \times \vec{B})]dv \tag{18}$$

This expression gives the net total force acting on the volume V; it is equal to the rate of change of the total momentum, electromagnetic and mechanical.

The forces exerted by the field on charges and neutral matter give rise to an increase in the mechanical momentum so that $\vec{F} = d/dt\ \vec{P}_m$. This force can be determined if the right-hand side of Eq. (18) can be split into two parts to be identified with P_m and P_e. However, the analysis of the problem may become difficult since this separation is not an obvious task to carry out. That is why the literature on the subject shows that several hypotheses have been suggested to define what part of the total momentum is associated with the strictive forces in the medium producing mechanical deformation.

In order to simplify the analysis of the mechanical forces arising from the interaction of a travelling electomagnetic wave with a material medium, it will be assumed that the medium is homogeneous, devoid of free-charges and of mechanical elasticity, and that steady-state conditions apply.

Under these assumptions, Eq. (18) reduces to an expression for the rate of change of electromagnetic momentum giving two force components which are represented by the terms $\vec{J} \times \vec{B}$ and $\partial/\partial t(\vec{D} \times \vec{B})$. As $\vec{J} \times \vec{B}$ is independent of time variations, it may be interpreted as the force that would arise in an equivalent DC field; on the other hand, $\partial/\partial t(\vec{D} \times \vec{B})$ can be interpreted as the force associated with the travelling electromagnetic wave.

It is then clear that to calculate the mechanical forces on the material medium, the time rate of change of the electromagnetic momentum has to be known.

5. Forces Due to Electromagnetic Energy and Velocity Gradients in a Material Medium

The total energy density due to the electric and magnetic fields is given by

$$W = \frac{1}{2}(\varepsilon E^2 + \mu H^2) \qquad (19)$$

This expression represents the instantaneous energy stored in the electromagnetic field per unit area. According to the theory of relativity, mass m and energy W are equivalent and must obey the relation

$$m = \frac{W}{c^2} \qquad (20)$$

Also, if mass m moving with velocity v has a rest mass m_r, the following relation must be valid:

$$m = \frac{m_r}{\sqrt{1-(v/c)^2}} \tag{21}$$

The binomial expansion of the right-hand side of Eq. (21) will result in the expression

$$m = m_r + m_k = m_r + m_r\left[\frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{3}{8}\left(\frac{v}{c}\right)^4 + \frac{15}{48}\left(\frac{v}{c}\right)^6 + \ldots\right] \tag{22}$$

m_k, the kinetic mass, is a function of the velocity v; it vanishes when v = 0.

Assuming that Eq. (20) can be applied to moving electromagnetic energy and its equivalent mass, then the energy W can be split into a rest energy W_r and an energy of relative motion W_k

$$W = W_r + W_k = (m_r + m_k)c^2 \tag{23}$$

Consider now a uniform plane wave with an angular frequency ω and a phase shift constant β in a medium with conductivity σ. The phase velocity v of the travelling wave is given by

$$v = \frac{\omega}{\beta} = \left(\frac{2}{\mu\varepsilon[\sqrt{1+(\sigma/\omega\varepsilon)^2} + 1}\right)^{1/2} \tag{24}$$

If the wave comprises an energy density $W_a = \frac{1}{2}(\varepsilon E^2 + \mu H^2)$, then the rate of flow of energy per unit area is:

$$s = \frac{1}{2}(\varepsilon E^2 + \mu H^2)v = W_a v = EH \qquad (25)$$

The magnitude of the electromagnetic momentum density $\vec{p} = \vec{D} \times \vec{B} = \mu\varepsilon(\vec{E} \times \vec{H})$ associated with the moving energy W_a is

$$p = \mu\varepsilon EH = \frac{EH}{1/\mu\varepsilon} = \frac{EH}{v_o^2} = \frac{W_a v}{v_o^2} \qquad (26)$$

where $v_o = 1/\sqrt{\mu\varepsilon}$.

The mass-energy equivalence principle applied to the energy W_a would be expressed by the relation $m_a = W_a/v_o^2$. Note that the energies W_a and W_k must not be considered the same quantities. W_k is an additional energy of motion for a rest energy W_r and it translates to equivalent mass in terms of c^2.

For a transverse electromagnetic wave progressing in the +Z direction, the electromagnetic momentum associated with an element of thickness δZ and unit cross-sectional area is given by the expression

$$\delta p = \frac{W_a v}{v_o^2}\,\delta Z \qquad (27)$$

As force is the time rate of change of momentum, the corresponding elementary force would be

$$\delta F = \frac{d}{dt}(\delta p) = \frac{1}{v_o^2}\left(v\,\frac{dW_a}{dt} + W_a\,\frac{dv}{dt}\right)\delta Z \qquad (28)$$

From basic calculus

$$\frac{d}{dt} = \frac{\partial}{\partial Z}\frac{\partial Z}{\partial t} = v\frac{\partial}{\partial Z}$$

so that

$$\delta F = \left(\frac{v^2}{v_o^2}\frac{\partial W_a}{\partial Z} + \frac{W_a v}{v_o^2}\frac{\partial v}{\partial Z}\right)\delta Z \tag{29}$$

The integration of Eq. (29) allows then the computation of the total force per unit surface area and if the time averages of the force, $\overline{F}$, and energy, $\overline{W}$, are considered, the expression for the total time average force per unit surface area will be

$$F = \frac{v^2}{v_o^2}\int\frac{\partial \overline{W}_a}{\partial Z}\,dZ + \frac{1}{v_o^2}\int\overline{W}_a v\frac{\partial v}{\partial Z}\,dZ \tag{30}$$

Eq. (30) denotes the presence of energy and velocity gradients and since mechanical forces can only operate on a material medium, these energy gradients must arise either from dissipation producing heat or from a progressive space change in the stored energy. Also, a space change in the propagation velocity yields a force component as it is the case when power is reflected at an interface between two different media.

As stated before, the gradients of energy in a source-free medium resulting in a rate of change of the

electromagnetic momentum, may be produced in two distinct ways:

(i) by conversion of energy to heat,

(ii) by transfer of stored energy to or from the material medium.

In the source-free conditions assumed, any additional energy put into the material as the wave propagates through it, whether to provide for heat losses or a change in the stored energy, must come from the empty space which the material occupies since conservation of energy requires a constant value of its total content. Any such transfer of energy from space to material medium is associated with a force acting in the +Z direction, that is, it is a positive force. Then, the dissipation of energy as heat or a progressive increase of the stored energy must produce a positive force.

The resultant force produced in a material medium through the process described is called radiation pressure.

6. Energy Gradients – A More Detailed Analysis

For the conditions assumed, it follows from Maxwell's equations that

$$\frac{\partial E_x}{\partial Z} = - \frac{\partial B_y}{\partial t} \tag{31}$$

$$- \frac{\partial H_y}{\partial Z} = \sigma E_x + \frac{\partial D_x}{\partial t} \tag{32}$$

Also, the expression $W_a = \frac{1}{2}(\varepsilon E_x^2 + \mu H_y^2)$ may be written as

$$W_a = \frac{1}{2}(\varepsilon E_x^2 + \mu H_y^2) \tag{33}$$

Combining Eqs. (31) and (32) with the partial derivative of Eq. (33) with respect to Z, an expression for the gradient of the total energy in the electromagnetic field can be readily obtained

$$\frac{\partial W_a}{\partial Z} = \frac{\partial}{\partial Z}(\frac{1}{2}\varepsilon E_x^2 + \frac{1}{2}\mu H_y^2)$$

$$= -\mu\sigma E_x H_y - \varepsilon E_x \frac{\partial B_y}{\partial t} - \mu H_y \frac{\partial D_x}{\partial t} + \frac{1}{2}(E_x^2 \frac{\partial \varepsilon}{\partial Z} + H_y^2 \frac{\partial \mu}{\partial Z}) \tag{34}$$

Under the assumptions being considered here, i.e., charge-free medium and unit cross-sectional area, it should be noted that the right-hand side of Eq. (34) corresponds to Eq. (18) and therefore it gives the rate of change of the total momentum including the part arising from strictive forces producing mechanical deformation which, however, is assumed to be negligible in the present discussion.

Since the medium is homogeneous the last term on the right side of Eq. (34) vanishes.

When the fields have a sinusoidal time variation, which is normally the case, the time average of the energy gradient corresponding to Eq. (34) is easily computed using

complex wave functions. Thus, representing the field components as complex amplitudes including both the phase factor and the space dependence of the wave function, the result is

$$\frac{\partial W_{al}}{\partial Z} = -\mu\sigma \mathrm{Re}(\tfrac{1}{2}E_x H_y^*) - \mu\varepsilon \mathrm{Re}(E_x^* \frac{\partial H_y}{\partial t}) - \mu\varepsilon \mathrm{Re}(H_y^* \frac{\partial E_x}{\partial t}) \qquad (35)$$

where Re and the asterisk stand, respectively, for the real part and complex conjugate of the quantities. But noting that

$$\mathrm{Re}(E_x^* \frac{\partial H_y}{\partial t}) = - \mathrm{Re}(H_y^* \frac{\partial E_x}{\partial t}) \qquad (36)$$

it happens that Eq. (35) reduces to the form

$$\frac{\partial W_{al}}{\partial Z} = -\mu\sigma \mathrm{Re}(\tfrac{1}{2}\, E_x\ H_y^*) \qquad (37)$$

This time average of the gradient of the total energy per unit volume is associated with conversion to heat, a fact that is a consequence of the finite conductivity σ of the medium; as this gradient is a negative quantity, it represents a progressive transfer of energy to the material with a gradient $+\frac{1}{2}\mu\sigma \mathrm{Re}(E_x\ H_y^*)$ and so the corresponding force is positive.

Eq. (37) does not reveal any gradient of stored energy in the material; therefore it is necessary for

further analysis to show the possibility of the existence of such a gradient in the medium responsible for a supplementary force.

Representing the permittivity and permeability of free space respectively by ε_o and μ_o, the instantaneous energy stored in the material medium alone per unit volume is given by

$$W_{a2} = \frac{1}{2}(\varepsilon - \varepsilon_o)E_x^2 + \frac{1}{2}(\mu - \mu_o)H_y^2 \tag{38}$$

Differentiating Eq. (38) with respect to Z, taking its time average using complex notation and assuming a homogeneous medium with $\mu = \mu_o$, the final expression for the gradient of stored energy would be

$$\frac{\partial \overline{W}_{a2}}{\partial Z} = \mu(\varepsilon - \varepsilon_o)\,\mathrm{Re}\left(\frac{1}{2}\, H_y^* \,\frac{\partial E_x}{\partial t}\right) \tag{39}$$

In the case of a pure travelling wave this gradient will be negative [Ref. 5] because as the field progresses through the medium it attenuates due to its finite conductivity. The gradient represents then energy progressively coming out of the medium and the force produced will be negative.

So, in general, it appears that a force component may result from a rate of change of electromagnetic momentum associated with stored energy.

7. Calculation of Radiation Pressure in an Infinite Homogeneous Medium

Eq. (30) is the basic equation for the calculation of the forces produced in a medium with characteristics μ, ε and σ, due to a rate of change of electromagnetic momentum associated with a travelling wave moving in the positive Z direction with a constant velocity v. Since the velocity is constant, the second term on the right-hand side of Eq. (30) is zero and so this equation becomes for a thickness d,

$$\overline{F} = \frac{v^2}{v_o^2} \int_0^d \frac{\partial \overline{W}_a}{\partial Z} dZ \tag{40}$$

$\partial \overline{W}_a/\partial Z$ can be decomposed into two gradients of energy according to Eqs. (37) and (38) so that, using the appropriate signs to yield positive forces, $\partial \overline{W}_a/\partial Z$ can be written as

$$\frac{\partial \overline{W}_a}{\partial Z} = - \frac{\partial \overline{W}_{a1}}{\partial Z} + \frac{\partial \overline{W}_{a2}}{\partial Z} \tag{41}$$

and Eq. (40) becomes

$$\overline{F} = \overline{F}_1 + \overline{F}_2$$

$$= \frac{v^2}{v_o^2} \int_0^d [\mu\sigma \mathrm{Re}(\tfrac{1}{2}E_x H_y^*) + \mu(\varepsilon - \varepsilon_o)\mathrm{Re}(\tfrac{1}{2}H_y^* \frac{\partial E_x}{\partial t})]dZ \tag{42}$$

where $\overline{F}_1$ and $\overline{F}_2$ are the average forces resulting from the gradients of energy associated, respectively, with dissipation as heat and with stored energy.

a. Forces in a Medium Extending from Z=0 to Z=∞

For a finite conductivity medium extending from Z=0 to Z=∞, the total power associated with a wave is absorbed which means that the overall stored energy is zero. Thus, no net component of force $\overline{F}_2$ can be expected. When this situation occurs, Eq. (42) takes the form

$$\overline{F} = \overline{F}_1 = \frac{v^2}{v_o^2} \int_0^\infty \mu\sigma \operatorname{Re}\left(\frac{1}{2} E_x H_y^*\right) dZ \tag{43}$$

The quantity $\operatorname{Re}(\frac{1}{2} E_x H_y^*)$ is the time average value of the instantaneous power density $\overline{s}$ carried by the wave which attenuates with Z at a rate determined by α, the attenuation coefficient of the medium; it can be shown [Ref. 6] that $\overline{s}$ is given by

$$\overline{s} = \operatorname{Re}\left(\frac{1}{2} E_x H_y^*\right) = \frac{E_{xo}^2}{2|\eta|} \cos\theta \, e^{-2\alpha Z} = \overline{s}_1 e^{-2\alpha Z} \tag{44}$$

where E_{xo} is the value of E_x at Z = 0 and $|\eta|$ is the magnitude of the complex intrinsic impedance of the medium $\eta = |\eta| \underline{/\theta}$.

Then the force coming from energy dissipation is given by

$$\overline{F} = \overline{F}_1 = \frac{v^2}{v_o^2} \int_0^\infty \mu\sigma \overline{s}_1 e^{-2\alpha Z} = \frac{v^2}{v_o^2} \mu\sigma \left(\frac{\overline{s}_1}{2\alpha}\right) \tag{45}$$

and since $v = 2\alpha/\mu\sigma$

$$\bar{F}_1 = \frac{v}{v_o^2} \bar{s}_1 \tag{46}$$

b. Forces Due to Reflection in the Medium

If the wave being propagated through the medium finds a partially reflecting surface, then a certain percentage $\bar{s} = \bar{W} v$ of the power it carries and its associated electromagnetic momentum, suffer a complete reversal and so the second term on the right side of Eq. (30) is no longer zero. This means that a force of reflection will be produced

$$\bar{F}_3 = \frac{1}{v_o^2} \int_{-v}^{+v} \bar{s} \frac{\partial v}{\partial Z} dZ = \frac{2\,\bar{s}\,v}{v_o^2} \tag{47}$$

For the particular case of a lossless medium ($\sigma = 0$), the fact that $v = v_o$ allows a further simplification of Eq. (47) and since $\mu = \mu_o \mu_r$ and $\varepsilon = \varepsilon_o \varepsilon_r$

$$\bar{F}_3 = \frac{2\bar{s}}{v_o} = \frac{2\bar{s}}{c} \sqrt{\varepsilon_r \mu_r} \tag{48}$$

c is the velocity of light in free space, ε_r and μ_r are respectively the dielectric constant and relative permeability of the medium outside the reflecting surface.

As seen from Eq. (48), the force due to reflection is directly proportional to $\sqrt{\varepsilon_r}$, the index of refraction of the medium.

8. Rate of Change of Electromagnetic Momentum for A DC Field and its Corresponding Force

When frequency approaches zero, the velocity v approaches zero too and the same happens to the electromagnetic momentum as can be seen from Eq. (26), for example.

Consider the material medium immersed in a DC field. If a rest energy W_b analogous to W_r is considered, it may be translated to equivalent mass m_b using the velocity c.

As a logical extension to this case of 'DC flow of energy', momentum may be then expressed by the relation

$$p = m_b c = (\frac{W_b}{c^2}) = \frac{W_b}{c} \tag{49}$$

So, even in the case of a DC field, the concept of an electromagnetic momentum responsible, when absorbed by the medium, for a mechanical force, is still valid. Without going in much detail, it seems natural to accept the idea of a smooth transition from wave propagation to DC transmission. It should be noted that this electromagnetic momentum density concept is equivalent to the one described by Eq. (4).

It will be shown next that under DC conditions, the production of force on the material medium via conversion of electrical energy to heat is still maintained.

W_b/c represents the volume density of electromagnetic momentum for a DC field in a homogeneous medium. When the

field components are transverse to the direction Z, the electromagnetic momentum associated with an element of thickness δZ and unit cross-sectional area is

$$\delta p = \frac{W_b}{c}\,\delta Z \tag{50}$$

so that the corresponding elementary force is

$$\delta F = \frac{d}{dt}(\delta p) = \frac{d}{dt}\left(\frac{W_b}{c}\,\delta Z\right) = \frac{dW_b}{dZ}\,\delta Z \tag{51}$$

By integration, the total force per unit surface area is given by

$$F = \int \frac{\partial W_b}{\partial Z}\,dZ \tag{52}$$

For time-invariant field components, the displacement current density $\partial D_x/\partial t$ vanishes and so Eq. (32) becomes

$$-\frac{\partial H_y}{\partial Z} = \sigma\,E_x \tag{53}$$

Using Eqs. (31) and (53), Eq. (34) reduces to

$$\frac{\partial W_b}{\partial Z} = -\mu\sigma E_x H_y \tag{54}$$

For a thickness d of the medium, the force per unit area arising from dissipation of power, according to Eq. (52), is given by

$$F = \mu\sigma \int_0^d E_x H_y \, dZ \tag{55}$$

Obviously, no gradient of stored energy contributes to this mechanism of production of force.

9. Summary of Important Results

The discussion of the forces arising in a material medium under the action of an electromagnetic field, was carried out in a simplified version and results in a few important relations.

It was shown that the process responsible for the production of such mechanical forces, is a rate of change of electromagnetic momentum associated with moving energy or its equivalent mass.

Three different mechanisms may be concurrent in the production of the resultant force, each one being, however, a consequence of distinct causes originating a change in momentum. These three major mechanisms described before are repeated here along with the corresponding equations.

a. Change in Momentum Due to Energy Gradients

Energy gradients in the medium may be produced in two distinct ways giving rise to different force components:

(i) By conversion of energy to heat

$$\overline{F} = \frac{v^2}{v_o^2} \int_0^d \mu\sigma R_e(\frac{1}{2}E_x H_y^*) \, dZ \tag{56}$$

(ii) By transfer of stored energy to or from the medium

$$\overline{F} = \frac{v^2}{v_o^2} \int_0^d \mu(\varepsilon-\varepsilon_o) R_e(\frac{1}{2}H_y^* \frac{\partial E_x}{\partial t}) \, dZ \tag{57}$$

b. Change in Momentum Due to Velocity Gradients

A force component can also arise if, for a given power ($W_a v$), there is a change in propagation velocity

$$\overline{F} = \frac{1}{v_o^2} \int \overline{W}_a v \frac{\partial v}{\partial Z} \, dZ \tag{58}$$

B. TRANSIENT SURFACE HEATING

1. Simple Thermoacoustic Source

Another process of the conversion of electromagnetic to acoustic energy to be described now, is the so-called thermal stress generation by transient surface heating of an elastic medium.

This is a known process since the end of the 19th century but only recently, due to improvements in high power laser technology, it was possible to make an experimental verification of the complex theory governing this mechanism.

The solution of the related problems require a certain number of assumptions so that the mathematical models used apply only to the specified conditions.

The transient heating of the surface of a body may be produced in several ways, namely, by electron bombardment or by electromagnetic energy absorption. The latter process is the one to be considered here, that is, the conversion to an elastic wave of that portion of the incident electromagnetic energy absorbed by the medium.

It will be adopted a one-dimensional treatment of the problem and assumed an uniform heating at and near the surface of a semi-infinite absorbing medium as well as an incident electromagnetic intensity uniformly distributed.

Under these conditions, a temperature gradient normal to the surface will be produced. This gradient is characterized by the physical properties of the medium and by the nature of the incident flux. The consequent thermal expansion gives rise to a production of strains in the medium which, in turn, leads to the generation of stress waves propagating away from the heated surface.

The time profile of the stresses and the amplitude of the waves produced for a given absorbed power density, depend upon the acoustic boundary conditions at the illuminated interface, that is, upon the elastic constraints applied to the heated surface. These boundary conditions may, in general, be divided into two types: stress-free

and constrained by contact with another body. The stresses and wave amplitudes obtained in the stress-free condition may be smaller than those obtained under constrained heated surface.

2. <u>Equation of Motion</u>

Consider that the plane $x = 0$ in a cartesian coordinate system is the surface of the semi-infinite elastic medium being illuminated with uniformly distributed electromagnetic energy. Part of this energy will be reflected at the interface but the remaining will penetrate the medium. If α and I_o represent, respectively, the attenuation constant of the medium and the radiation intensity at the surface, then the radiation intensity $I(x)$ at a distance x from the plane $x = 0$ is given by the familiar exponential law

$$I(x) = I_o \, e^{-\alpha x} \tag{59}$$

This means that the electromagnetic wave is being absorbed by the medium as it progresses through it.

This absorption of energy causes a temperature rise at and near the surface and, as a result of this effect, a strain ε_x is produced

$$\varepsilon_x = \frac{\partial u(x,t)}{\partial x} = a\theta(x,t) \tag{60}$$

where u(x,t) is the x component of particle displacement, a is the coefficient of linear thermal expansion and $\bar{\theta}(x,t)$ is the temperature rise above the uniform initial temperature. Recalling that the problem is being treated in one dimension

$$\frac{\partial u(x,t)}{\partial y} = \frac{\partial u(x,t)}{\partial Z} = 0 \tag{61}$$

therefore the strains ε_y and ε_Z are also zero.

In order to produce the strain ε_x in the absence of heating, a stress σ_x would have been necessary

$$\sigma_x = - Ba\theta \tag{62}$$

where B is the bulk modulus of elasticity.

In the presence of both heating and stress, it can be shown [Ref. 7] that the corresponding stress-strain relationship for the case of negligible lateral inertia and shear, exhibited by liquid media for example, is given by

$$\sigma_x = B\varepsilon_x - Ba\theta = B\frac{\partial u}{\partial x} - Ba\theta \tag{63}$$

The resulting equation of motion for the semi-infinite body is then expressed in the form

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_x}{\partial x}$$

$$= \frac{\partial}{\partial x} \left(B \frac{\partial u}{\partial x} - Ba\theta\right) \tag{64}$$

where ρ is the density of the medium. Noting that the modulus of elasticity and the velocity v of the compressional wave propagation in the medium, are related by

$$B = \rho v^2 \tag{65}$$

eq. (64) becomes

$$\frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - a \frac{\partial \theta}{\partial x} \tag{66}$$

This is the nonhomogeneous wave equation to be solved for the elastic particle displacement.

3. Temperature Distributions

In order to solve eq. (66), the temperature distribution function $\theta(x,t)$ has to be known.

For the case of an electromagnetic wave, if it was assumed that the whole energy was absorbed in the plane $x = 0$, relatively simple approximate temperature distributions could be obtained. However, absorption of energy actually takes place in a region of finite thickness which is related to the skin depth. If this fact is taken into account, the temperature distributions are obtained through a more difficult analysis but they will be accurate.

A particular temperature distribution is examined here: it is the one resulting from a constant input electromagnetic intensity of short time duration, suffering an exponential attenuation in the medium. It is assumed that most of the input energy degenerates to heat and that the process is adiabatic.

If radiation of intensity I_o hits the interface in the time range $t = 0$ to $t = T_*$ a heat source in the region $x > 0$ will be created having a heat production rate determined by eq. (59). The resultant heat flow can be considered one-dimensional as long as the penetration depth α^{-1} is much less than the diameter d of the surface receiving heat.

A solution for this temperature distribution, where the heat flow is supposed to be in the positive x direction, was achieved by Carslaw and Jaeger [Ref. 8] but the final form of the function $\theta(x,t)$ is a rather complicated expression.

Under certain conditions, the solution just mentioned may be simplified. In fact, an approximation can be found [Ref. 7] for the case of intense heating of media having low thermal conductivity, such as liquids, and for durations of the radiation pulses from 10 to 50 nsec. This approximation is expressed by

$$\theta(x,t) = \frac{\alpha I_o t e^{-\alpha x}}{J \rho C} \tag{67}$$

where J and C are, respectively, the mechanical equivalent of heat and the specific heat of the medium.

This type of heating can be produced using, for example, a Q-switched laser with output in the range 50-100 MW/cm^2. The temperature rises so obtained, although rapid, do not exceed tens of degrees but the experienced temperature gradients would be extremely large. As an example [Ref. 7], it could be expected in water colored by a dye to give $\alpha = 10^2 cm^{-1}$, temperature gradients as high as $7.5x10^3$ °C/cm and temperature rates of change of the order of $2.5x10^9$ °C/sec at the surface during the laser pulse.

If nonmetalic media is considered, the temperature decay for t > T is a slowly varying function of time, becoming significant only for t greater than milliseconds. Also, the elastic transients produced through this process, propagate away from the heated place in periods of time as short as microseconds. So, it can be assumed that for t > T

$$\theta(x,t) = \frac{\alpha I_o T e^{-\alpha x}}{J\rho C} \tag{68}$$

Equations (67) and (68) represent then the source functions to be used in eq. (66) in order to determine the equations for displacement and stress.

4. Solution of the Equation of Motion For Two Boundary Conditions

a. Constrained surface (rigid boundary)

Substituting eqs. (67) and (68) in eq. (66), the resultant wave equations during and after the heating pulse are

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} + \begin{cases} v^2 W t e^{-\alpha x} , & t < T , \\ v^2 W T e^{-\alpha x} , & t > T \end{cases} \tag{69}$$

where

$$W = \frac{a\alpha^2 I_o}{J\rho C} \tag{70}$$

The solution of these equations for the initial conditions

$$\left.\frac{\partial u}{\partial t}\right|_{x,0} = u(x,0) = 0 \tag{71}$$

and the appropriate boundary condition for a constrained surface

$$u(0,t) = 0 \tag{72}$$

would give the expressions for the particle displacement $u(x,t)$ in the time ranges $T < t < x/v$, $x/v < t < x/v + T$ and $t > x/v + T$ [Ref. 7]. Once $u(x,t)$ is known, $\partial u/\partial x$ can be substituted in eq. (63) to obtain the expression for the stress in the medium as a function of time and distance from the surface. Since the condition $\alpha^{-1} << d$ is met, an elastic plane wave is then launched from the surface and propagates away from it.

Again, due to the extreme complexity of the equations for displacement and stress actually obtained, it is desirable to try some sort of valid simplification to render them more suitable for specific applications. This is possible if the values of x are chosen such that $\alpha x \gg 1$, that is, the points x should be those far from the source. Then, introducing a reduced time variable $\tau = t - x/v$, the approximate expressions for the stress would take the form

$$\sigma(x,t) = \frac{va I_o}{2JC} \cdot \begin{cases} (1 - e^{-\alpha vT}) e^{\alpha v\tau} & , \quad \tau<0 \ , \\ [2 - e^{-\alpha v\tau} - e^{\alpha v(\tau-T)}] & , \quad 0<\tau<T, \\ (e^{\alpha vT} - 1) e^{-\alpha v\tau} & , \quad \tau>T \ . \end{cases} \tag{73}$$

From these equations it can be observed that the maximum stress occurs at $\tau = T/2$ and its value is given by

$$\sigma_{max} = \frac{va I_o}{2JC} (1 - e^{-\alpha vT/2}) \tag{74}$$

which approaches the limiting value of $vaI_o/2JC$ for $\alpha vT/2 \gg 1$.

b. Free surface (pressure-release boundary)

The boundary condition at the stress-free surface is now

$$\sigma(0,t) = B \frac{\partial u(0,t)}{\partial x} - Ba\theta(0,t) = 0 \tag{75}$$

or, equivalently,

$$\frac{\partial u(0,t)}{\partial x} = a\theta(0,t) = \begin{cases} \dfrac{a\alpha I_o t}{J\rho C} \quad , & t<T \quad , \\ \\ \dfrac{a\alpha I_o T}{J\rho C} \quad , & t>T \quad , \end{cases} \tag{76}$$

Under this condition and following a procedure similar to the one used for the constrained surface, it is possible to get a set of approximated equations describing the stress at observation points far from the heated surface, $\alpha x >> 1$, and in terms of τ

$$\sigma(x,t) = \frac{vaI_o}{2JC} \cdot \begin{cases} (1 - e^{-\alpha vT})\ e^{\alpha v\tau} & \tau<0 \quad , \\ [e^{-\alpha v\tau} - e^{\alpha v(\tau-T)}] \quad , & 0<\tau<T \quad , \\ (1 - e^{\alpha vT})\, e^{-\alpha v\tau} \quad , & \tau>T \quad . \end{cases} \tag{77}$$

These equations tell that maximum stresses having the value

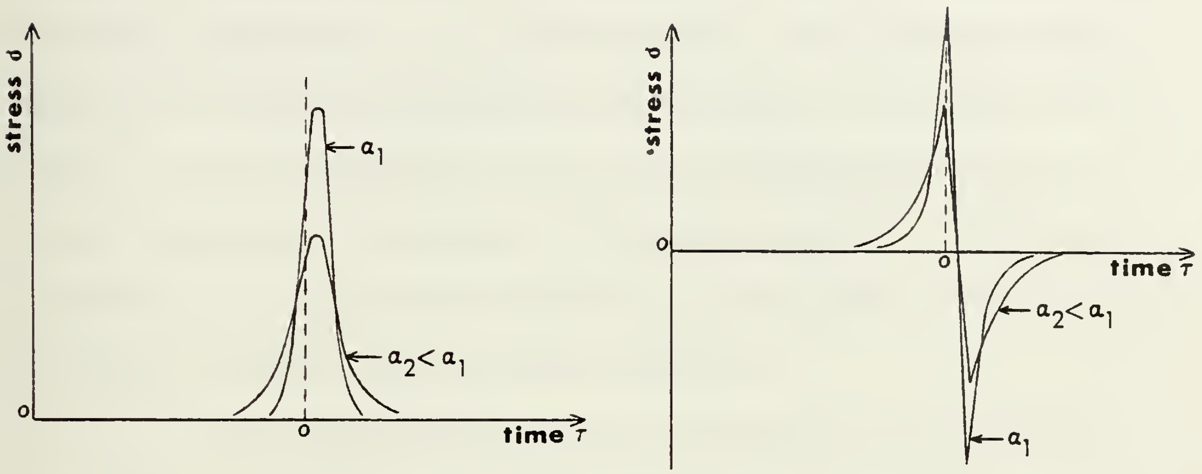
$$\sigma_{max} = \frac{vaI_o}{2JC}\ (1 - e^{-\alpha vT}) \tag{78}$$

appear at $\tau = 0$ and $\tau = T$ and approach a limiting value of $vaI_o/2JC$ for $\alpha vT >> 1$.

5. Stress Waves in the Medium

Solving eqs. (73) and (77) for various values of the parameter αvT, a plot of stress versus reduced time can be obtained.

Typical curves obtained in this manner would look like those shown in Fig. 1.



(a) (b)

Fig. 1. Typical stress-time plots for constrained surface (a) and stress-free surface (b)

The constrained surface situation gives rise to stress consisting of a unidirectional compressive pulse, while in the free surface case the stress suffers an abrupt change from compression to tension during the time T.

To give an idea about the theoretical magnitude of the resultant stresses, the use of the free-surface expressions for the case of a 100 MW, 30 nsec laser pulse, illuminating water with $\alpha = 100\ cm^{-1}$, would lead to produced stresses with peaks of ±20 atm.

As expected from the theory, the peak amplitude of the pulse decreases and its duration increases as the attenuation constant of the medium is reduced.

High power laser pulses may be then responsible for the production of thermal effects in media. The rapid changes in temperature occuring at a surface absorbing rapidly changing amounts of energy is due to the fact that thermal conduction is a slow process. The elastic waves detected at points far from the surface of a medium as a result of the absorption of an electromagnetic pulse of high frequency, will have a frequency content consisting primarily of frequency components associated with the pulse envelope rather than with the carrier.

A few experiments on the generation of acoustic signals in liquids by laser pulses, had been performed recently and the results seem to confirm the theory behind the transient heating process of elastic wave generation.

E.F. Carome, N.A. Clark and C.E. Moeller [Ref. 9], using a laser beam from a 0.1 Joules, 50 nsec single pulsed ruby laser, verified the validity of eqs. (73) and (77) for the case of transient heating of water surface. The measurements were performed with various concentrations of Prussian blue in order to change the attenuation constant of the water. The shape of the pulses obtained, closely approximate those predicted by theory. As observed, an increase in α, that is, an increase in the absorptivity of the medium, increased the elastic wave amplitude. This result indicates that the produced elastic waves could not have been generated through a process of radiation pressure by reflection at the surface since an increase in absorptivity decreases the amount of reflected energy.

6. Efficiency of Energy Conversion

The conversion efficiency N of this process is defined as the ratio of the energy of the propagated elastic wave E_a to the energy of the incident electromagnetic wave E_e

$$N = \frac{E_a}{E_e} \tag{79}$$

As may be guessed, this process is characterized by a very low efficiency.

For both free and constrained surface conditions, the energy E_a is a function of the area through which the elastic wave is propagated and the area over which the electromagnetic wave is incident. Assuming that both areas have the same value, the resultant expressions for N [Ref. 7] are, for the case of free-surface

$$N = \frac{va^2 I_o}{2\rho C^2 J^2} \cdot F_1(\alpha vT) \tag{80}$$

where

$$F_1(\alpha vT) = \frac{1 - e^{-\alpha vT} - \alpha vTe^{-\alpha vT}}{\alpha vT} \tag{81}$$

and for the case of constrained surface

$$N = \frac{va^2 I_o}{2\rho C^2 J^2} \cdot F_2(\alpha vT) \tag{82}$$

where

$$F_2(\alpha vT) = \frac{\alpha vTe^{-\alpha vT} + 3e^{-\alpha vT} + 2\alpha vT - 3}{\alpha vT} \qquad (83)$$

In these expressions I_o is in watts per square centimeter while all the other quantities are in the cgs system of units.

A plot of the functions F_1 and F_2 versus αvT, allow an easier and quicker numerical calculation of N. These plots show that the maximum value of F_1 is 0.3 for $\alpha vT = 2.0$ and that F_2 approaches a maximum of 2.0 for large values of αvT.

As an illustration of a numerical calculation of the efficiency showing its dependence on the properaties of the medium, the value $N = 1.83 \times 10^{-11} I_o F_1(\alpha vT)$ is obtained for water and $N = 6.14 \times 10^{-9} I_o F_1(\alpha vT)$ for carbon tetrachloride, both for the free surface condition.

A final remark about the functional dependence of N on the variable I_o should be made. From the anaysis of eqs. (80) and (82) it seems apparent that efficiency can be increased without limit as I_o is increased. However, this cannot be a true statement since beyond a certain value of input power the assumed model of stress generation is no longer valid. Some limitations of this model result from the following facts. First, the thermal characteristics of the medium impose limits on the radiation intensity since phenomena like boiling in liquids or melting and vaporization in solids may occur. Stresses still continue to be generated

although by a different process. Secondly, since at high intensities the thermal and elastic properties of the medium become temperature dependent, the formulas used before are no longer applicable. And finally, for significant values of efficiency, an appreciable fraction of the input energy is lost by radiation and so the temperature distribution assumed cannot be used.

7. Thermoacoustic Arrays

The way the conversion mechanism has been treated here through a simplified thermodynamic model emphasizing the process of conversion of laser light to acoustic energy in liquids, may be considered an engineering approach of great practical interest.

This phenomenon can be analyzed in a more general sense to include the treatment of a wider range of specific cases with different types of media and electromagnetic sources.

R.M. White [Ref. 10] in a theoretical and experimental work analyzed the conversion process for several temperature distributions due to different input energy fluxes.

The generation of low frequency sound from the absorption of light has become a subject of growing importance with potential significance in underwater acoustics. Westervelt and Larson [Ref. 11] in a recent theoretical formulation, predicted the development of highly directive sound beams in water by means of thermalization of modulated laser light. The generation of this highly directional

acoustic wave results from an extension of the concept of a simple thermoacoustic source to that of a thermoacoustic array, that is, a series of thermoacoustic sources where pressure fields combine to give a directional effect.

A theoretical summary of the mechanism leading to the preduction of such a wave, which propagates perpendicular to the axis of the laser beam, is given next.

Assume that a laser radiates an amplitude modulated beam of cross sectional area S_o propagating in the +z direction as shown in Fig. 2.

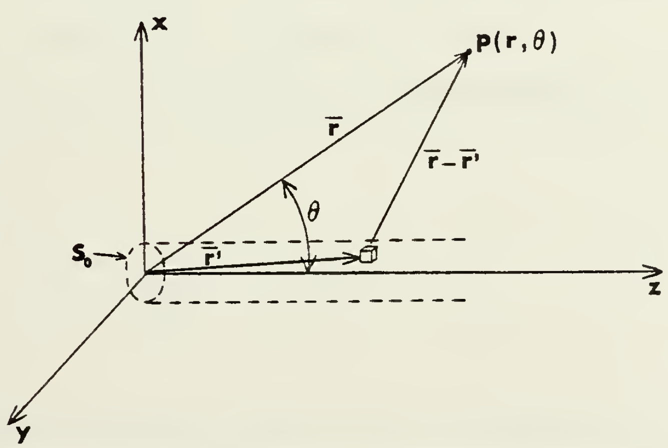


Fig. 2. Geometry of Laser Excitation

Due to the absorption of energy in the medium, each elemental volume experiences a rise in temperature which, in turn, causes an oscillatory expansion in the thermal volume. If the beam illuminates the surface of a medium of low thermal conductivity, the solution of the wave equation

for the pressure distribution resulting from the exponential absorption of energy is

$$p = -\frac{j\alpha\omega a I_o}{4\pi C} e^{-j\omega t} \int e^{-\alpha z} \frac{e^{j\omega/v|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} d^3r' \qquad (84)$$

where ω is the laser angular modulation frequency, a the logarithmic coefficient of thermal expansion, C the specific heat per unit area, I_o the output intensity of the laser, α the attenuation coefficient for the laser beam and v the sound velocity. This result was obtained for a source function representing the energy per unit volume per unit time absorbed, given by $I_o \exp(-\alpha z - j\omega t)$.

Looking for a far field approximation, r has to obey the condition

$$r \gg \frac{\omega}{\alpha^2 v} = \frac{k}{\alpha^2} \qquad (85)$$

So, simplifying and performing the integraion on eq. (84), the acoustic pressure field radiated by this mechanism turns out to be

$$p = -\frac{j\alpha\omega a P_o}{4\pi r C} \cdot \frac{e^{jkr-j\omega t}}{\alpha + jk\cos\,\theta} \qquad (86)$$

where P_o is the laser output power. Averaging eq. (86) over one period, the resulting acoustic intensity would be given by

$$I = \frac{1}{2\rho C} \cdot \left(\frac{\alpha\omega a P_o}{4\pi r C}\right)^2 \cdot \frac{1}{\alpha^2 + k^2\cos^2\theta} \quad (87)$$

In this equation ρ represents the density of the medium.

The maximum value of eq. (87) occurs at $\theta = \pi/2$ and the angle at which the intensity is reduced to one half its maximum is

$$\theta_{1/2} = \frac{\pi}{2} \pm \frac{\alpha}{k} \quad (88)$$

The total acoustic power generated when the limit condition $k/\alpha >> 1$ applies, is given by

$$w = \frac{\alpha\omega a^2 P_o^2}{16\rho C^2} \quad (89)$$

The radiation pattern given by eq. (87) is symmetric about the Z axis and has a sharp maximum at $\theta = \pi/2$ in the limit $\alpha << k$.

Equations (88) and (89) show that when α increases giving rise to more intense thermal expansions, larger acoustic power outputs are produced, but when α decreases, the length of the array increases and so the directivity is improved.

On the other hand, for a given value of α, increasing the modulation frequency ω, the beam may be made as narrow as desired. An increase in conversion efficiency is also achieved as ω is increased.

The array acts then like a broadside radiator where directivity is proportional to the attenuation coefficient.

All these theoretical results were verified and validated by experiments performed by T.G. Muir, C.R. Culbertson and J.R. Clynch [Ref. 12]. These experiments showed further that for applications of laser excited arrays to ocean acoustics, high-power lasers in the green-blue region are required in order to obtain extremely narrow acoustic beams.

C. STIMULATED BRILLOUIN SCATTERING

1. Brillouin Scattering

Sound waves in a medium may also be created from the interference of two intersecting laser beams. This effect has been verified experimentally [Refs. 13,14] and its detailed theoretical formulation may be found in the literature [Refs. 15,16].

A scattering phenomenon can be defined for the purposes of the present discussion as a process whereby a physical system extracts energy from a beam of light and re-emits this energy at the same or at a different wavelength, in the same or different directions from the incident beam.

Scattering may occur in a number of different situations but among all the forms it can take, Brillouin scattering is the one to consider as far as conversion of electromagnetic to acoustic energy is concerned. This type

of scattering is an interaction process between acoustic waves at microwave frequencies and electromagnetic waves at optical frequencies.

It has been accepted that an electromagnetic field has both a particle and a wave nature. Such a statement leads to the definition of photon as a particle characterized by zero mass, energy $h\nu$ and wave vector $\vec{k}$. Similarly, the treatment of lattice modes of vibration in material media, leads to the phonon concept, a particle having zero mass, energy $h\nu$ and momentum $\overrightarrow{\bar{h}g}$ where ν is the phonon frequency, $\vec{g}$ the phonon wave vector, $\bar{h} = h/2\pi$ and h the Planck's constant.

The acoustic waves involved in the production of Brillouin scattering, usually range from 10^6 to 10^{12} Hz and can be externally introduced into a material medium, can be already present in the material in the form of thermally activated phonons or can be stimulated in the medium by the light beam itself.

2. <u>Bragg Reflection</u>

Consider that a sound wave at angular frequency ω_m is traveling upward in a cubic block of a material medium, as shown in Fig. 3 in a cross section perspective.

When a plane wave of light of angular frequency ω_o hits the left face of the block in a direction such that its wavefronts are perpendicular to the acoustic wavefronts, an interaction process will take place within the material.

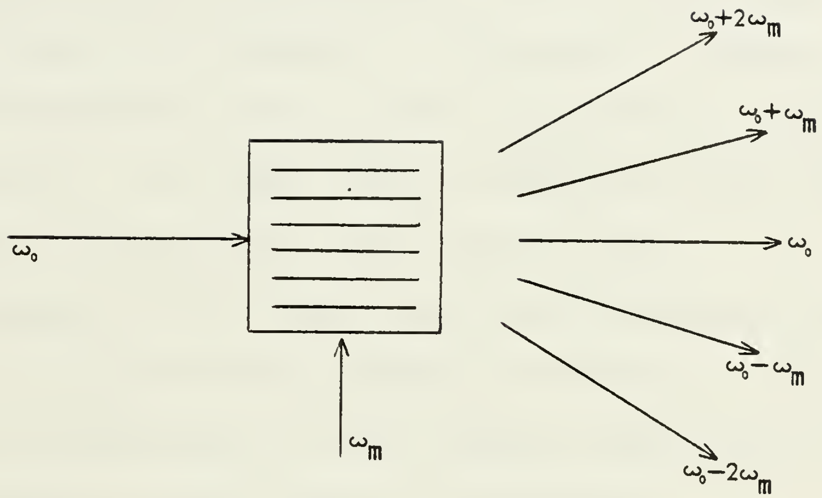


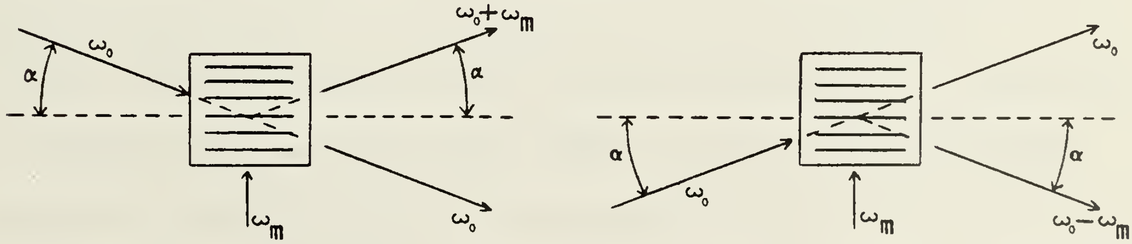
FIG. 3 — The Debye-Sears Effect

The results of this scattering process, called Debye-Sears effect, is similar to what happens when a carrier frequency ω_o is phase modulated with a single tone ω_m, that is, light at angular sideband frequencies ω_o, $\omega_o \pm \omega_m$, $\omega_o \pm 2\omega_m$, etc., will emerge from the right face of the block. If λ and Λ represent, respectively, the light and sound wavelengths, then the wavefronts associated with the first upper and lower sidebands $\omega_o \pm \omega_m$, will be tilted by an angle $\theta \approx \pm\lambda/\Lambda$ with respect to the undeflected carrier light ω_o. Similarly, for the nth order sidebands, the deflection angle will be $\pm\, n\lambda/\Lambda$.

The output intensity of the various sidebands is the same not only because their amplitudes have to obey Bessel functions behaviour, but also because a destructive interference occurs in the medium. For a fixed ω_m, the extent in which each sideband is 'destroyed' depends upon its frequency as well as upon the physical dimensions of the material. So, this type of scattering will diffract just a small amount of the total input light.

Suppose now that the input ray is rotated to make an angle $\alpha \simeq \lambda/2\Lambda$ with the acoustic wavefronts. The process developed is equivalent to that occuring when the incident ray was parallel to the acoustic wavefronts.

As the first upper sideband still makes an angle λ/Λ with the undeflected output light, the interaction process for this sideband may be though of as a simple reflection of light from the acoustic wavefronts. This effect is depicted in Fig. 4.



(a) (b)

FIG. 4 — The Bragg Effect Geometry

Moreover, this sideband is the only one suffering a constructive interference in the medium and therefore it is the only one with significant intensity.

The angle of incidence α such that $\alpha \approx \lambda/2\Lambda$, or more precisely

$$\alpha = \sin^{-1} \frac{\lambda}{2\Lambda} \qquad (90)$$

is called Bragg angle. Clearly, a Bragg angle can be defined for each output frequency.

Due to this mechanism, the Bragg effect allows the selection of a particular mode of output light by a simple control of the direction of the incident light, that is, by forcing light and sound to meet at the appropriate angle.

It can be shown that at exactly the Bragg angle, the scattering attains its maximum.

3. <u>Production of Sound Waves</u>

When two light beams with frequencies ω_1 and ω_2 and wave vectors $\vec{K}_1$ and $\vec{K}_2$ intersect in a medium, an induced electrostrictive pressure wave of frequency $|\omega_1-\omega_2|$ and wave vector $\vec{K}_1-\vec{K}_2$ may be produced. If the angle of intersection of the light beams is such that the phase velocity $\omega_1-\omega_2/|\vec{K}_1-\vec{K}_2|$ of the pressure wave is the same as the velocity of the sound in the medium, then a traveling wave will be produced. It is verified that the angle satisfying this condition is precisely twice the Bragg

angle. It should be noted that this effect can only occur for an input beam energy above a well defined threshold.

This mechanism of interaction of intense light beams leading to the production of coherent sound is called stimulated Brillouin scattering.

Consider now the situation shown in Fig. 4 where light and sound intersect at the Bragg angle. The incident and scattered light wavefronts intersect at twice the Bragg angle and so they should form an interaction pattern giving rise to stimulated Brillouin scattering. Thus, additional sound at ω_m is launched from the interaction region.

It turns out that in the case of Fig. 4(a), the produced (additional) and original sound waves are in opposite phases causing a progressive decrease of the sound amplitude (deamplification). On the other hand, if the situation shown in Fig. 4(b) occurs, the produced and original sound waves are in phase and amplification of sound is then verified.

The stimulated Brillouin scattering mechanism may be reduced to the following picture: if the input energy is high enough, instability conditions are created within the medium and, even when no input sound is present, production and buildup of sound at $\omega_m = |\omega_1 - \omega_2|$ starting from thermally activated phonons, will be verified. Or, in other words, the elctrostrictive mixing of two light beams gives rise to amplification of thermal sound in the medium.

Brillouin scattering is a classic example of parametric interactions in traveling wave systems where the power transfer between the waves may be calculated on the basis of either the Manley-Rowe relations or simple photon-phonon energy conservation arguments. It is important to realize that the case of stimulated Brillouin scattering gives an example where the principle of parametric oscillations can perform a useful function, namely, the output energy at the lower frequency is in the form of sound as contrasted to the electromagnetic energy of the pump.

It has been proved experimentally [Ref. 13] that the observed sound cannot be the result of thermal shock in the medium. In fact, removing one of the light beams without affecting the total input light power, it can be observed that sound is no longer produced. That is, the signal observed at the beat frequency of the two light beams is indeed optically induced sound.

III. EXPERIMENTAL PROCEDURE

A. APPARATUS DESCRIPTION AND RESULTS

1. Radiation Pressure Experiment

In the first experiment performed it was intended to observe the forces of radiation pressure developed in fresh water by a plane electromagnetic wave.

Due to the mismatch between air and water, part of the energy is reflected back at the interface and part of it is transmitted through the water which is assumed here to be devoid of mechanical elasticity and of free charges.

To perform the experiment, a portable radar model AN/PPS-6(XE-6) was used as a source of electromagnetic field. Some characteristics of interest of this radar are:

pulse repetition rate --------------- 2 KHz

frequency --------------------------- 9.3 GHz

pulse length ------------------------ 0.5 μsec

peak power -------------------------- 100 W

average power ----------------------- 47 mW

power supply requirements ----------- 13 V at 1.5A

The power supply chosen was a POWER DESIGNS INC., model 3240. Appropriate pieces of coaxial cable and rectangular waveguide both assumed lossless, were used to connect the radar output to a horn antenna, a SPERRY MICROLINE model

56 x 1. According to its calibration curves an absolute gain of 15.8 db can be expected at 9.3 GHz. The antenna was oriented to face the surface of the water with the axis nearly perpendicular to it so that a transverse uniform plane wave could be assumed at the illuminated surface.

In order to detect the pressure waves produced in the water, an hydrophone type LC-10 was used. This is a small device about 5.4 cm long having a circular cross section with diameter about 0.96 cm. The useful steady state bandwidth of this acoustic detector extends from 0.1 Hz to 120 KHz. The typical free-field voltage sensitivity within this range lies between -105 and -110 db relative to 1 V/μbar. The actual hydrophone used was the LC10 serial #2122 exhibiting a flat free field frequency response between 1 KHz and 9 KHz at -110 db.

The free field voltage sensitivity, or microphone sensitivity level ML, is defined by the relation

$$ML = 20 \log \frac{V/P}{1V/\mu bar}$$

where V is the amplitude of the voltage output, P is the pressure amplitude of the acoustical field and 1V/μbar is the reference sensitivity.

The LC10 typical directivity pattern in the plane perpendicular to the axis is almost circular. During the measurement, this device was held in the water in a position parallel to the surface and in the same vertical of

the transmitting horn antenna. The hydrophone depth could be varied since it was bonded to a horizontal plastic rod 45 cm long which in turn was attached to a vertical metallic rod; this assembly could be moved vertically in front of a tape-measure stuck on a fixed wall.

Through a 25 ft. coaxial cable the hydrophone output signal was fed to a HEWLETT PACKARD voltage amplifier model 465A capable of either 20 or 40 db of amplification.

Two different schemes were tried to measure the forces of radiation pressure. In the first, a cubic wood box about 6 ft. high was placed over one of the acoustic tanks in Spanagel Hall Room 025. The box was covered inside with a good absorber of electromagnetic energy; the transmitting equipment was placed on its top side where a small hole was made to let the horn antenna go inside. The hydrophone assembly was stuck to one of the vertical wood walls. The distance between the horn and the surface of the water was fixed at about 1 m.

The output of the voltage amplifier was connected to the input of a TEKTRONIX oscilloscope type 422 either directly or through a very narrow band-pass filter. This filtering action was achieved through the operation as a narrow band-pass filter of a wave analyzer HEWLETT PACKARD model 302A.

Due to the considerable environmental random noise amplitude and to the extreme weakness of the signal produced by the energy conversion mechanism, no effect could

be observed in the oscilloscope. No signal component at about 2 KHz appeared when the output signal was passed through the band-pass filter, neither any signal resembling the envelope of the transmitted waveform could be seen over the noise when the output of the voltage amplifier was fed directly to the oscilloscope. The magnitude of the signal to be expected is computed later in the theoretical computations section.

In the second scheme used it was intended to take advantage of a crosscorrelation or autocorrelation process.

Instead of making the detection in the big acoustic tank mentioned before, a small portable water container was used. It was covered inside with styrofoam to minimize acoustic reflections; its internal dimensions were approximately 26 x 30 x 67 cm. This time, the horn antenna was held at about 10 cm above the surface of the water.

A tunable envelope detector was installed in the wave-guide in order to get the envelope of the transmitted waveform to be crosscorrelated with the received signal via hydrophone. A correlation and probability analyser SAICOR model SAI-43A with capability of performing either auto or crosscorrelation seemed to be a suitable equipment to use in front of the amplified signal. The output of this analyzer may be displayed in any ordinary oscilloscope.

When the autocorrelation of the envelope of the transmitted waveform was tried, no result was obtained since the detected pulse was so narrow that even in the

fastest sampling mode of the analyser no autocorrelation could be achieved on this waveform. So, instead of trying to detect the presence of the signal in the noise using crosscorrelation it was decided to make simply the autocorrelation of the noiselike waveform at the output of the voltage amplifier. The use of two voltage amplifiers in series became necessary in order to overcome a well defined input threshold level since only after this signal level is achieved the equipment can perform properly.

A non-expected waveform was displayed in the oscilloscope after the autocorrelation operation. Rather than being associated with the acoustic waveform expected, it was simply the result of the autocorrelation of a 60 Hz power line interference corrupted with random noise. It could be observed that the same autocorrelation waveform was exactly obtained whether the radar was transmitting or not.

As a conclusion of these experiments and according to the theoretical calculations, such a low input electromagnetic energy is not enough to give detectable results after suffering this extremely weak conversion process.

2. <u>Transient Surface Heating Experiment</u>

In the second group of experiments performed the fresh water was assumed an elastic medium. If results were eventually obtained by using a high power pulsed laser they could be attributed mainly to a thermally shocked

interface rather than to the simultaneous but much weaker radiation pressure effects.

Two types of lasers were used in this part of the experimental work as sources of electromagnetic energy.

The first one was the Nitrogen pulsed laser existing in Bullard Hall Room 217. This device is capable of delivering in normal operating conditions as much as 80 kw peak power. Its pulse repetition rate can be varied but 20 pulses per second was the one used. The pulse width is about 10 μsec and the wavelength is 0.3371 μ.

The detection apparatus was again the hydrophone, voltage amplifier and oscilloscope.

Unfortunately the output light of this source has not the shape of a narrow beam; instead it diverges too much and in order to focus as much light as possible several lenses had to be used. A mirror was also used to deflect to a vertical orientation the horizontal output beam. As a result of all these effects and characteristics the input energy at the surface of the water was not enough to generate a detectable signal at the oscilloscope display. Due to improper shielding of the laser, electromagnetic interference at the pulse repetition frequency was picked up also random noise and power line interference were observed.

No conversion effect was then detected through this scheme.

The second laser used was a single pulse laser existing in the Laser Laboratory, Building 223. This is a

high energy Neodymium Glass Rod laser operating at the wavelength 1.06 μ and its output light is highly directional. The beam was deflected to the vertical orientation using a low absorption prism. It is possible to vary its output energy and values up to 12 Joules can be safely obtained. However with the control knobs set at a fixed position it is observed that the output energy do not stay constant from shot to shot. The extent of this variation can be appreciated next in Fig. 6.

Through the use of a beam splitter a portion of the output beam was diverged toward a photo detector whose output voltage signal was used to measure the energy of each output pulse. Another photo detector excited by the visible light of the laser flash tube was used for scope triggering purposes. This triggering action took place about 0.1 msec earlier than the beginning of the laser pulse. So, there is a time delay of about 0.1 msec between the starting of the sweep and the beginning of the laser pulse that should be considered when interpreting Fig. 6. Two TEKTRONIX oscilloscopes were used: a memo-scope type 564B to measure the energy of each pulse and a 545A with single sweep capability to observe and photograph the detected signal with a polaroid TEKTRONIX oscilloscope camera system. A sketch of the experimental setup is shown in Fig. 5.

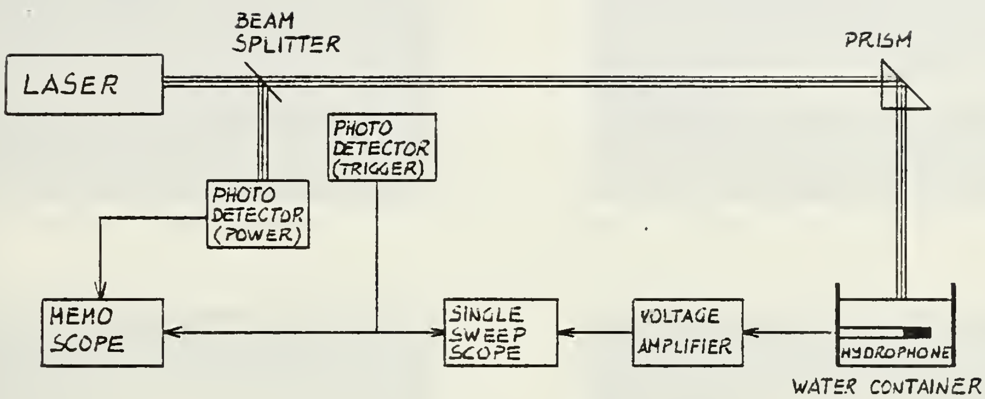
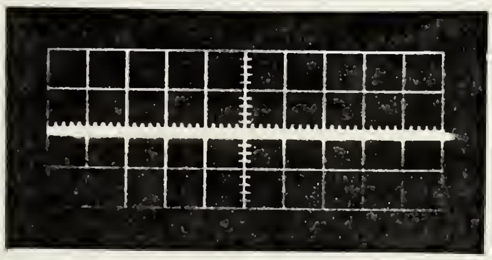


FIG. 5 — Experimental Setup

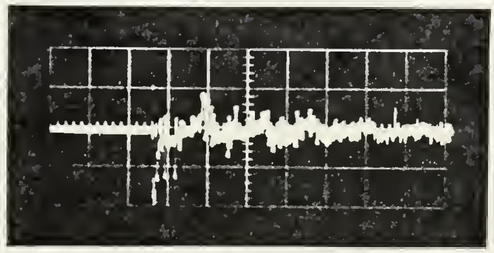
The experiment was performed with sea water to decrease the penetration depth of the laser beam. In fact at 2.83×10^{14} Hz, which is the laser frequency, the penetration depth for sea water with productivity $\sigma = 4$ mhos/m is about 1.19 cm while for fresh water it is orders of magnitude greater. Thus, the use of sea water would guarantee a total absorption of the electromagnetic energy within the limited system used.

The hydrophone was not exactly in the same vertical of the laser beam to prevent the possibility of damage caused by the intense radiation.

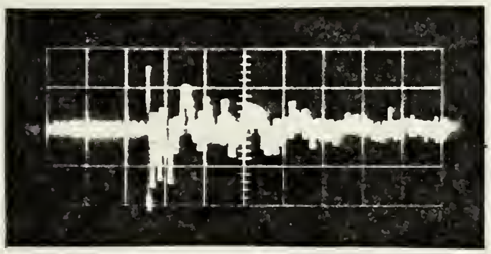
To make sure that the signal obtained after each shot was not the result of electromagnetic interference, a flat piece of wood was placed over the container to prevent the radiation from reaching the surface of the water.



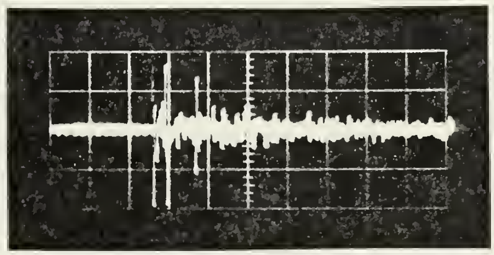
(a) 6.6J, no input



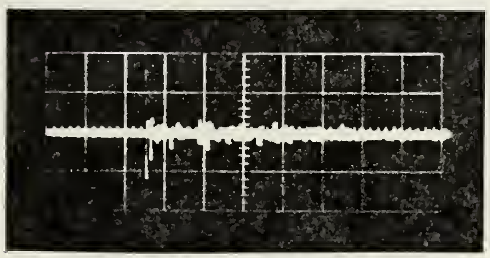
(b) 6.9J, free, 5 cm



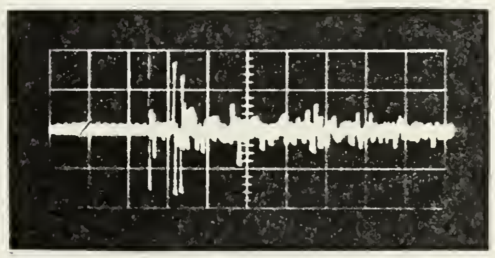
(c) 6.6J, free, 5 cm



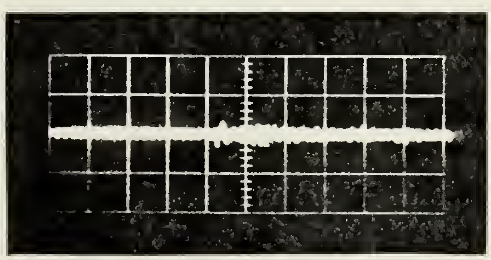
(d) 6.8J, constrained, 5 cm



(e) 6.3J, free, 10 cm



(f) 7.1J, constrained, 10 cm



(g) 5.7J, free, 17 cm

(h) 6.1J, free, 17 cm

FIG. 6 — The output energy, water surface condition and hydrophone depth are shown for each shot. Horizontal scale is 0.1 msec/div for all pictures. Vertical scale is 5v/div except for picture (h) where it is 2v/div.

Picture 6(a) was taken under these conditions and as it can be seen no signal was recorded; this shows clearly that the signals obtained before came really from a surface heating conversion process.

Fig. 6 presents the results of a series of shots for various surface conditions and hydrophone depths. The voltage amplifier was kept at 20 db for all of them. As mentioned before although the laser controls were at a fixed position the output energy was not constant and values ranging from 5.7 to 7.1 Joules were obtained.

From pictures 6(b) to (f) it can be seen that the time delay between the starting of the laser pulse and the beginning of the detected signal is about the same whether the hydrophone was at 5 or 10 cm depth and much bigger than the time that sound in water takes to travel 5 or 10 cm. Obviously this is due to the facts that heating is not instantaneous and that the energy is not only abosrbed at the surface that causes the thermal effect and the sound production to take place but over a considerable region, the extent of which is determined by the penetration depth.

Pictures (g) and (h) were taken with the hydrophone much farther from the beam vertical than in all the others and at 17 cm from the small surface area illuminated. Here the effect of a greater distance from the shocked region is apparent since the time delay is now greater.

Pictures (d) and (f) were taken under constrained surface conditions. As predicted by the theory the signal

obtained had a higher amplitude; it also happened however that the energy in both cases was higher than in the free surface case and so it is not clear in which extent the increase in amplitude was determined by the surface constraint. It should be noted that a circular glass plate of about 80 cm^2 was used touching the water to constrain its surface which means that just a small percentage of it was constrained.

Clear is the fact that in general the shape of the waveforms obtained is not close to the theoretical predictions made before. It is not difficult to accept such a different behavior since the assumed theoretical conditions were not fulfilled in this experiment, namely the observation point was not in the far field and the heating was not at or near the surface. Moreover, it is known that the envelope of the output pulse of this laser system is not a neat rectangle but exhibits instead two peaks and has a shape and duration varying from pulse to pulse. Also the relatively long plastic rod supporting the hydrophone may easily get into vibration when subjected to the acoustic pressure and acoustical reflections from the walls are likely to occur.

To appreciate the pressure field peak amplitude consider for example fig. 6(c) where a peak voltage amplitude of about 10 V was obtained. Taking into account the 20 db of signal amplification and using eq. (91) it can be computed that the pressure amplitude of the produced acoustical field is 3.16 bars or 3.12 atmospheres.

B. THEORETICAL COMPUTATIONS

Theoretical calculations related to the radiation process are now made to give a further insight on the problem and to allow a better understanding of the difficulty in getting experimental results at low input power levels.

Assume that the medium under consideration is sea water with conductivity σ = 4 mhos/m, relative permittivity $\varepsilon_r = 80$ and relative permeability $\mu_r = 1$.

The reflection coefficient R for a plane electromagnetic wave incident normally on the surface of the water is given by

$$R = \frac{E_r}{E_i} = \frac{\eta - \eta_o}{\eta + \eta_o}$$

where E_r and E_i are respectively the reflected and incident E field strengths, $\eta_o = \sqrt{\mu_o/\varepsilon_o} = 377\Omega$ is the intrinsic impedance of free space and

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon_o\varepsilon_r}}$$

is the intrinsic impedance of the water. When the frequency of the radiation is f = 9.3 GHZ then

$$\omega = 2\pi f = 58.43 \times 10^9 \text{ rad/sec},$$

$$\eta = 42.02 \angle 2.76^\circ ,$$

$$R = 0.799 \angle -0.623^\circ .$$

This means that only 20.1% of the E field will penetrate the water.

If the incident electromagnetic energy is from a radar with output peak power P_t = 100 W and if a horn antenna with gain G_t = 15.8 db = 38.02 is at r = 1 m above the surface of the water, then the power density at the interface is

$$P_i = \frac{P_t G_t}{4\pi r^2} = 302.55 \text{ W/m}^2 = \frac{E_i^2}{\eta_o}$$

so $E_i^2 = \eta_o \times 302.55$ or $E_i = 337.73$ V/m. Then

$$337.73 \times 0.799 = 67.88 \text{ V/m}$$

will penetrate the water.

The attenuation constant of the water is given by

$$\alpha = \omega\left[\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right)\right]^{1/2} = 84.142 \text{ Np/m}$$

The penetration depth $\delta = 1/\alpha$ is then equal to 1.188 cm. So, as far as absorption of energy is concerned, the medium may practically be considered as extending from z = 0 to z = ∞. Thus the components of the radiation pressure force are in this case the force given by eq. (46) due to energy dissipation of 20.1% of the incident energy and the force given by eq. (48) due to the reflection at the interface of 79.9% of that energy.

From eq. (46) the average force due to dissipation of the transmitted energy is

$$\bar{F}_1 = \frac{v}{v_o^2} \times \frac{E_{xo}^2}{2|\eta|} \cos\theta$$

where $|\eta| = 42.02\ \Omega$, $\theta = 2^\circ\!.76$ and $E_{xo} = 67.88$ V/m; v is given by eq. (24) and $v_o = 1/\sqrt{\mu\varepsilon}$.

Computing the values of v and v_o^2 it turns out that

$$v = 33.48 \times 10^6 \text{ m/sec,}$$
$$v_o^2 = 11.23 \times 10^{14} \text{ .}$$

The average force is then

$$\begin{aligned} F_1 &= 163.195 \times 10^{-8} \text{ N/m}^2 \\ &= 163.195 \times 10^{-7} \text{ dyne/cm}^2 \\ &= 163.195 \times 10^{-7}\ \mu\text{bars.} \end{aligned}$$

With this value for the average pressure field and knowing that the hydrophone sensitivity level is ML = -110 db relative to 1V/μbar, it is possible using eq. (91) to compute what would be the hydrophone output voltage

$$\begin{aligned} -110 &= 20 \log V/\bar{F}_1 \\ \log V/\bar{F}_1 &= -5.5 \\ V/\bar{F}_1 &= 3.16 \times 10^{-6} \\ V &= 0.516 \times 10^{-10} \text{ Volts.} \end{aligned}$$

From eq. (48) the average force due to energy reflection at the interface is given by

$$\overline{F}_3 = \frac{2\overline{s}}{c} \quad \sqrt{\varepsilon_r \mu_r}$$

where the factor $\sqrt{\varepsilon_r \mu_r}$ is now 1 since incidence and reflection of energy are taking place in free space, $c = 3. \times 10^8$ m/sec and

$$\overline{s} = \frac{E_{xo}^2}{2\eta} = \frac{269.85^2}{2 \times 377} = 96.577 \text{ W/m}^2$$

So

$$\overline{F}_3 = \frac{2\overline{s}}{c} = 64.385 \times 10^{-8} \text{ N/m}^2$$

$$= 64.385 \times 10^{-7} \text{ μbars}$$

Following the same procedure as before the output voltage would be

$$V = \overline{F}_3 \times 10^{-6} = 0.203 \times 10^{-10} \text{ Volts.}$$

The values just obtained for the output voltages due to both force components are so small that they cannot be easily seen in ordinary oscilloscopes. More sophisticated detection schemes could lead to positive results but as mentioned before even through a correlation process, it was not possible to detect the presence of the signal.

Electromagnetic energy has to be increased orders of magnitude if radiation pressure forces are to be easily displayed by detection equipment.

IV. CONCLUSIONS

According to theoretical predictions and experimental results, in order to get significant acoustic power, extremely high electromagnetic energy has to be used. The reason for this is the low efficiency that characterizes this type of energy conversion.

In practice, when the conversion process takes place, both radiation pressure and transient surface heating forces will be developed in the medium.

Thermal forces are by far more important since they are orders of magnitude larger than radiation pressure forces. The present status of laser technology has spurred interest in the possible applications of this conversion process. In the future, it is likely that there will be ocean applications of highly directive thermoacoustic arrays excited by modulated lasers.

Radiation pressure forces, although much smaller in magnitude and requiring careful design of detection schemes, higher power sources and highly sensitive devices, have been proposed to be used in several applications.

A. Ashkin [Ref. 17] observed that micron-sized particles in liquids and gas may be accelerated and trapped in a laser beam. The experiments were performed taking special care to avoid as much as possible thermal effects from the laser light so that the observed effects were due primarily to

radiation pressure forces. Chemical and laser pumping applications were suggseted on the basis of this effect.

H.E.M. Barlow points out in his paper [Ref. 4] the possibilities of the application of radiation pressure forces in power measuring instruments and in orientation behavior in satellites.

Much work has to be done yet in the field of the conversion of electromagnetic to acoustic energy until all useful applications are thoroughly explored.

As a final comment it should be noted that the Laser Laboratory at NPGS seems to have the appropriate material available to encourage further experimental work on the production of acoustic waves in media through transient surface heating.

APPENDIX A

DETAILED THEORETICAL CALCULATIONS

1. Penetration depth (δ) for sea water

$\sigma = 4$ mhos/m

$\varepsilon_r = 80$

$\varepsilon = \varepsilon_o \varepsilon_r = 708.32 \times 10^{-12}$ F/m

$\mu_r = 1$

a. For the radar frequency $f = 9.3$ GHZ

$\omega = 2\pi f = 2 \quad \times 9.3 \times 10^9 = 58.43 \times 10^9$ rad/sec

$$\mu\varepsilon = \mu_o \varepsilon_o \varepsilon_r = 4\pi \times 10^{-7} \times 8.854 \times 10^{-12} \times 80$$
$$= 8.901 \times 10^{-16}$$

$$\frac{\sigma}{\omega\varepsilon} = \frac{4}{58.43 \times 10^9 \times 708.32 \times 10^{-12}} = \frac{4}{41.3871} = 0.0966484$$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right)}$$

$$= 58.43 \times 10^9 \sqrt{4.4505 \times 10^{-16} \left(\sqrt{1 + 0.009341} - 1\right)}$$

$$= 58.43 \times 10^9 \sqrt{4.4505 \times 10^{-16} \times 0.0046596}$$

$$= 58.43 \times 10^9 \sqrt{0.020738 \times 10^{-16}}$$

$$= 84.142 \text{ Np/m}$$

$$\delta = \frac{1}{\alpha} = 0.01188 \text{ m} = 1.188 \text{ cm}$$

b. For the laser frequency

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{1.06 \times 10^{-6}} = 2.83 \times 10^{14} \text{ HZ}$$

$$\omega = 2\pi f = 17.78 \times 10^{14} \text{ rad/sec}$$

$$\frac{\sigma}{\omega\varepsilon} = \frac{4}{17.78 \times 10^{14} \times 708.32 \times 10^{-12}} = \frac{4}{1259392.96}$$

$$= 3.1761 \times 10^{-6}$$

$$\alpha = 17.78 \times 10^{14} \sqrt{4.4505 \times 10^{-16} \times 5 \times 10^{-12}}$$

$$= 83.873 \text{ Np/m}$$

$$\delta = \frac{1}{\alpha} = 0.01192 \text{ m} = 1.192 \text{ cm}$$

2. Reflection coefficient (R) for sea water for the radar frequency

$$R = \frac{E_r}{E_i} = \frac{\eta - \eta_o}{\eta + \eta_o}$$

$$\omega = 58.43 \times 10^9 \text{ rad/sec}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j158.43 \times 10^9 \times 4\pi \times 10^{-7}}{4 + j58.43 \times 10^9 \times 708.32 \times 10^{-12}}}$$

$$= \sqrt{\frac{j73425.3035}{4 + j41.3871}} = \sqrt{\frac{73425.3035\ e^{j90°}}{41.5799e^{j84°.48}}}$$

$$= 42.0224\ e^{j2°.76} = 41.974 + j2.0235$$

$$R = \frac{41.974 + j2.0235 - 377}{41.974 + j2.0235 + 377} = \frac{-335.026 + j2.0235}{418.974 + j2.0235}$$

$$= \frac{335.032e^{j0°.3461}}{418.979e^{j0°.2767}} = 0.799\ \underline{|-0°.623}$$

3. Radiation pressure force $\overline{F}_1$ due to energy dissipation

$$\overline{F}_1 = \frac{v}{v_o^{\ 2}} \times \frac{E_{xo}^2}{2|\eta|} \cos\theta$$

$$v = \frac{\omega}{\beta} = \sqrt{\frac{2}{\mu\varepsilon[\sqrt{1 + (\sigma/\mu\varepsilon)^2} + 1]}}$$

$$= \sqrt{\frac{2}{8.901 \times 10^{-16}\ [\sqrt{1 + (0.0966484)^2} + 1]}}$$

$$= \sqrt{\frac{2}{8.901 \times 10^{-16} \times 2.00466}} = 0.3348 \times 10^8 \text{ m/sec}$$

$$v_o^{\ 2} = \frac{1}{\mu\varepsilon} = \frac{1}{8.901 \times 10^{-16}} = 0.11235 \times 10^{16}$$

$$\overline{F}_1 = \frac{0.3348 \times 10^8}{0.11235 \times 10^{16}} \times \frac{67.88^2}{2 \times 42.02} \times 0.9988 = 163.195 \times 10^{-8} \text{ N/m}^2$$

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